

Reduction of Cabin Noise During Cruise Conditions by Stringer and Frame Damping

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Control of low-frequency cabin noise is a difficult problem in all commercial aircraft. The subject is analyzed in terms of the response of a pressurized fuselage structure subjected to broadband random pressure fluctuations. It is shown that in the so called "stiffness controlled" region, the structural response and noise transmission may be governed by the resonances of the stiffeners, with the skin acting like an attached mass. As a result, cabin noise at low frequencies may be reduced by application of constrained viscoelastic damping treatments on the stringers and frames of the fuselage.

Nomenclature

a, A_{st}	= stringer spacing and area of cross section
b	= frame spacing
C_{ws}	= warping constant of the stringer cross section, about the point of skin contact
E_{sk}, E_{st}	= Young's moduli of the skin and the stringer materials
f	= frequency
G_{st}	= shear modulus of the stringer material
h	= skin thickness
I_η, I_s	= stringer moments of inertia in bending and torsion
J_{st}	= St. Venant constant of uniform torsion for the stringer cross section
K_R	$= E_{st} (1 + i\eta_{st}) C_{ws} \beta^4 + G_{st} (1 + i\eta_{st}) J_{st} \beta^2 - \rho_{st} I_s \omega^2$
K_T	$= E_{st} (1 + i\eta_{st}) I_\eta \beta^4 - \rho_{st} A_{st} \omega^2$
n	= number of half waves between two frames
p	= cabin pressure differential
R	= fuselage radius of curvature
β	$= n\pi/b$
η_{sk}, η_{st}	= skin and stringer damping loss factors
ν	= Poisson's ratio of skin material
ρ_{sk}, ρ_{st}	= skin and stringer material densities

1. Introduction

CABIN noise in the mid- and high-frequency regions (above 600 Hz) can be adequately treated with skin damping tape, lead-vinyl sheets, and fiberglass insulation. However, these treatments are not very effective at low frequencies. For this reason, control of low-frequency cabin noise has remained a difficult problem in all commercial aircraft. It is anticipated that both the upper surface blown (USB) and the externally blown flap (EBF) STOL aircraft will have higher levels of low-frequency interior noise, because of the proximity of the engines to the fuselage and the generation of low-frequency noise components in USB and EBF environments.^{1,2} Control of low-frequency cabin noise in the energy efficient prop-fan aircraft is also likely to be a difficult task.

In Ref. 3, the possibility of reducing low-frequency cabin noise in an unpressurized aircraft by intrinsic tuning of the skin and stringers and subsequent applications of damping treatment on the stringers was discussed. It was shown that when the structure is intrinsically tuned, the structural

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response around the fundamental frequency of the individual skin bay is reduced significantly. It was also shown that a significant reduction of low-frequency structural response can be achieved by designing the structure so that the skin bay frequency is higher than the stringer frequency and then applying damping treatment on the stringer flanges. This was demonstrated by both laboratory and field test results. The results of this study therefore can be used to reduce low-frequency cabin noise and sonically induced stresses during takeoff.

During cruise, the in-plane loads due to cabin pressurization play a significant part in determining the fuselage structural response to noise and boundary-layer turbulence. The effect of pressurization on the transmission loss of a cylinder was considered by Koval.^{4,5} In Ref. 4 the fuselage was represented by an unstiffened shell. In Ref. 5 the effect of the stiffeners was considered by "smearing out" the stringers and frames. The role of the flexural deformation of the stringers and frames in controlling the "overall" modes of the fuselage at low frequencies was discussed only in qualitative terms in Ref. 6, and the effect of cabin pressurization was also not considered.

In the present study, the stiffeners are represented as discrete members with stiffness and mass properties. Two different structural models are considered, i.e., a periodic skin-stringer panel (Fig. 1) and a periodic frame-stiffened cylinder (Fig. 2). Both models are assumed to be under the influence of in-plane tensile loads due to cabin pressurization.

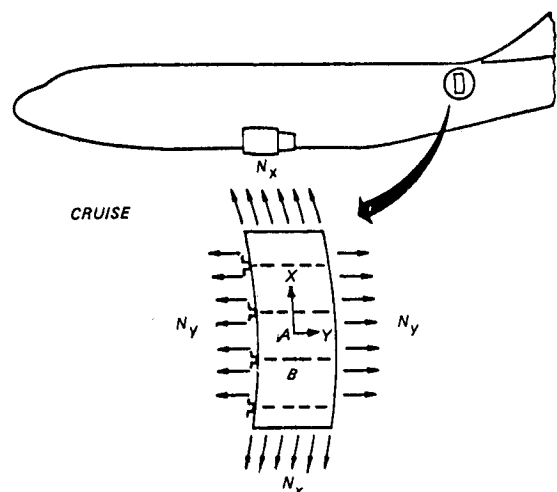


Fig. 1 Analysis of pressurized fuselage, low- to mid-frequency model ($200 \text{ Hz} < f < 1250 \text{ Hz}$).

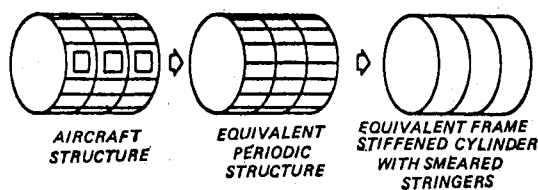


Fig. 2 Analysis of pressurized fuselage, low-frequency model ($f < 250$ Hz).

II. Structural Models

In the frequency range $200 \text{ Hz} < f < 1250 \text{ Hz}$, the structural model consists of an infinitely long periodic skin-stringer structure (Fig. 1) carrying in-plane loads due to cabin pressurization during cruise. The skin and the stringers are assumed to be simply supported at the frames. For the sake of simplicity, the skin-stringer model is assumed to be flat rather than curved. It has been observed that the natural frequency of a curved skin panel carrying in-plane tensile loads pR in the circumferential direction and $pR/2$ in the longitudinal direction is influenced primarily by the in-plane loads, the curvature itself playing a secondary role. For example, the fundamental natural frequency of a curved $22.8 \times 50.8 \times 0.16$ cm ($9 \times 20 \times 0.063$ in.) panel with a radius of 322.6 cm (127 in.) and a pressure differential of $58.2 \times 10^3 \text{ N/m}^2$ (8.45 psi) is calculated to be 610 Hz, with simply supported edges along the frames and clamped edges along the stringers. The corresponding frequency of a flat panel of the same dimensions, carrying the same in-plane loads, is found to be about 530 Hz. Thus, exclusion of the curvature has the effect of reducing the frequency by only about 13%.[†] The analysis was therefore carried out in terms of a periodically stiffened flat panel carrying in-plane tensile loads. This was advantageous because an existing computer program for analyzing flat periodic skin-stringer panels could be used with only a minor modification to include the effect of the in-plane loads. In order to compensate for the lack of curvature, an equivalent pressure differential was used, so that the natural frequency of the flat panel with in-plane loads $p_{eq}R$ and $p_{eq}R/2$ was equal to that of the curved panel with in-plane loads pR and $pR/2$ in the circumferential and longitudinal directions. In the above example, an equivalent pressure differential of $77.5 \times 10^3 \text{ N/m}^2$ (11.25 psi) increased the flat panel natural frequency to 610 Hz and this was used in the analysis.

In order to simplify the analysis, it was also assumed that the in-plane tensile load in the longitudinal direction due to cabin pressurization is entirely carried by the skin. In an actual fuselage, a part of this load is carried by the stringers. However, it was found that the increase in the stringer bending natural frequency due to this in-plane load is negligible ($< 2\%$) because of its inherent high stiffness. Consequently, the effect of the longitudinal load on the stringers was not considered. Also, in an actual fuselage, the longitudinal load carried by the skin is somewhat less than $pR/2$. However, the skin natural frequency is influenced primarily by the circumferential tensile load pR since it is the dominant term and, consequently, a small change in the longitudinal load causes a much smaller change in the skin panel natural frequency. For these reasons, it was felt that the above simplifying assumption about the skin carrying the entire longitudinal in-plane tensile load was justified.

To analyze the low-frequency ($f < 250$ Hz) noise transmission problem, a periodic frame-stiffened cylinder carrying in-plane loads due to cabin pressurization was considered (Fig. 2). Again, it was assumed that the circumferential load due to pressurization was carried entirely by the skin, since the frame in-plane bending natural frequencies

[†]It may be noted that the natural frequency of the same panel without carrying any in-plane load is about 178 Hz. This may be calculated by using Ref. 7.

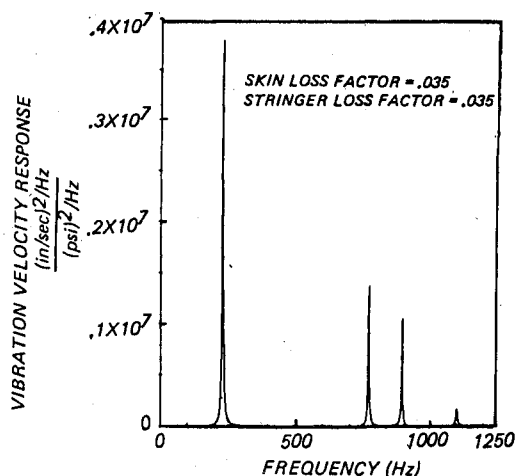


Fig. 3 Predicted velocity response of the baseline structure at the center of the skin bay (point A, Fig. 1).

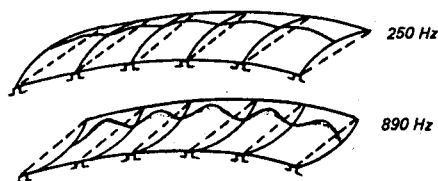


Fig. 4 Predicted modeshapes of the pressurized skin-stringer panel.

are not significantly altered by the hoop tension load carried by the frames.

III. Response of Pressurized Skin-Stringer Structures

During cruise, the aft section of a commercial airplane is excited by the boundary-layer pressure fluctuations and by jet efflux noise from the wing-mounted engines. Typically, a thick boundary layer develops in the aft section, with the spectrum of pressure fluctuations reaching a peak in the 250-350 Hz range. The combined excitation has convecting components both in the longitudinal and the circumferential directions. (If the boundary layer turbulence acts as the dominant source of excitation, it has strongly convecting components in the longitudinal direction.) The correlation of pressure fluctuations decays in the longitudinal and the circumferential directions. For a rigorous analysis of the structural response, it is necessary to decompose the excitation in terms of its wave number-frequency spectra, so that the response may be obtained in terms of all of the convected plane wave components. The response at any frequency may then be obtained by a summation over all possible values of the excitation wave number at that frequency. Examples of such calculations may be found in Refs. 8 and 9.

The method of predicting the response of a pressurized skin-stringer structure to a random pressure field convected in the longitudinal and circumferential directions is given in the Appendix.

IV. Results

Based on the equations described in the Appendix, a computer program for predicting the response of skin-stringer structures carrying in-plane tensile loads to broadband random excitation was developed. The structural data used corresponded to the aft section of a commercial airplane, with 0.16 cm (0.063 in.) thick skin, 22.8 cm (9 in.) stringer spacing, 50.8 cm (20 in.) frame spacing, $58.2 \times 10^3 \text{ N/m}^2$ (8.45 psi) cabin pressure differential, and 322.6 cm (127 in.) fuselage radius. The stringers were of top-hat cross section with 1.81

cm^4 (0.0436 in.^4) bending moment of inertia and 1.26 cm^2 (0.196 in.^2) cross-sectional area. In order to reduce the amount of computation, the excitation was considered to have a white noise spectrum with infinite trace velocities along the circumferential and longitudinal directions.† (The PSD of excitation was considered to be unity in all the computations.) During cruise, the actual excitation in an aircraft may be a combination of jet noise and boundary-layer pressure fluctuations, or it may be predominately due to the boundary-layer pressure fluctuations, depending upon whether the engines are mounted on the wings or in the tail section. Therefore, the response of a real aircraft structure will differ from what is predicted from this analysis. Nevertheless, certain essential conclusions regarding the mechanism of structural response may be drawn, even from the present simplified analysis.

Since the noise radiation from a structure is a function of its vibration velocity response, the predicted vibration velocity response at the center of each skin bay is shown in Fig. 3. For the baseline structure with no external damping treatment, a skin loss factor of 0.035 and a stringer loss factor of 0.035 were assumed. These are values typical of riveted skin-stringer structures. From Fig. 3 it may be seen that the spectrum is dominated by a peak at about 250 Hz. This peak is associated with a mode in which the skin and the stringers vibrate in phase, with one single half-wave in between two frames (Fig. 4). There is another mode at about 775 Hz that corresponds to a mode with three half-waves between the frames. Another mode appears at about 890 Hz. In this mode, there is again a single half-wave between the frames, but now the stringers and panels vibrate out of phase (Fig. 4). The mode at about 1100 Hz has five half-waves between the frames. There are also higher order modes at frequencies higher than 1250 Hz. Since the excitation is assumed to be highly correlated over a large structural area, the modes with an even number of half-waves between the frames do not appear in the response spectrum.

Figure 5 shows the predicted skin acceleration response spectrum. The peaks observed in Fig. 3 may also be identified in Fig. 5. The predicted acceleration response of the stringers is shown in Fig. 6. Comparing Figs. 5 and 6, it is seen that at frequencies above 500 Hz, the predicted skin response is generally higher than the stringer response. The skin response above 500 Hz is mainly governed by the resonant response of the pressurized, rectangular skin panel bounded by the stringers and frames. It is also seen that around 250 Hz the predicted skin and stringer responses approach each other.

Comparison with Existing Flight Test Data

In order to verify the predicted characteristics of the fuselage structural response, an existing set of flight test data¹⁰ was re-evaluated. Calculations similar to those described above were performed using the structural data for this particular aircraft, and the existence of a structural mode around 250 Hz, similar to that just described was predicted for this case. The PSD's of the acceleration responses of the skin and the stringers, obtained from the flight test of the pressurized aircraft, are shown in Fig. 7. During the flight test, the excitation consisted of a combination of noise from the engines and pressure fluctuations due to boundary-layer turbulence. Because of the various simplifying assumptions used in the analysis regarding the excitation and the structure, no direct comparison between the predicted and measured values could be made. However, certain similarities between the trends of the measured and predicted spectra may be observed.

Figure 7 shows that, above 500 Hz, the measured skin response is considerably higher than the stringer response. There are also several peaks in the skin response spectrum above 500 Hz. These indicate that above 500 Hz or so, the

†This is equivalent to assuming that the excitation field does not have any substantial phase variation over a large area.

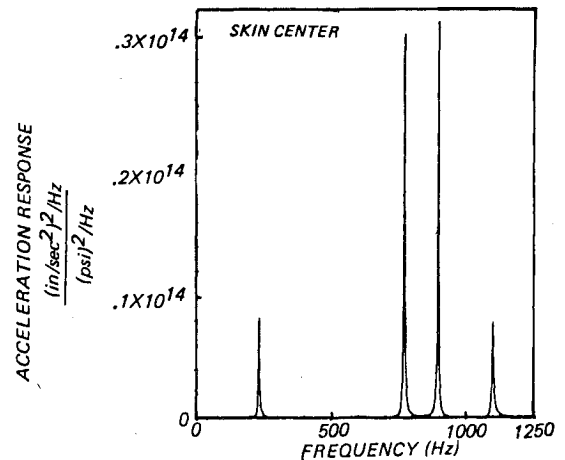


Fig. 5 Predicted acceleration response of the baseline structure at the center of the skin bay (point A, Fig. 1).

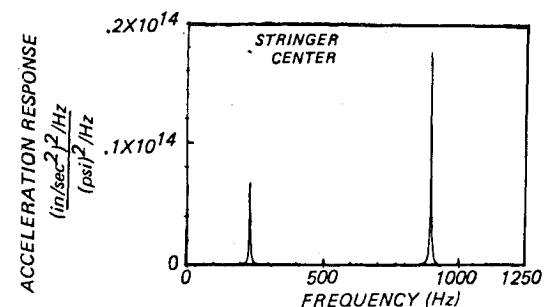


Fig. 6 Predicted acceleration response of the baseline structure at the stringer center (point B, Fig. 1).

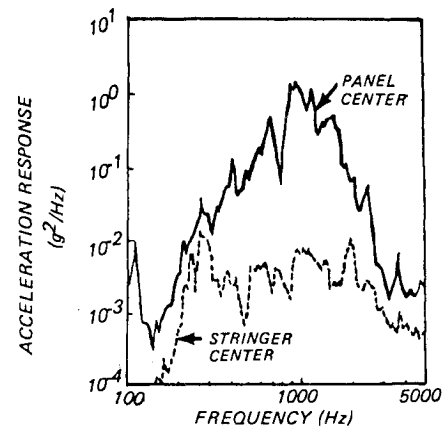


Fig. 7 Measured acceleration response at the skin and stringer centers.

skin vibration is largely governed by the resonant response of the pressurized rectangular skin panel bounded by the stringers and frames, as predicted by the analysis. It is also observed that around 250 Hz, the skin and stringer responses approach each other, as predicted by the analysis (compare Figs. 5-7).

Effect of Skin Damping

Figure 8 shows the predicted response spectrum of the center of the skin bay, when the skin loss factor is increased to 0.15 while the stringer loss factor is kept unchanged at 0.035. Comparing with Fig. 3, it is seen that skin damping is highly effective in reducing the response of a pressurized structure in the 500-1250 Hz range. However, the peak at 250 Hz is hardly affected by skin damping. In the midfrequency range, the response is essentially controlled by the bending modes of the

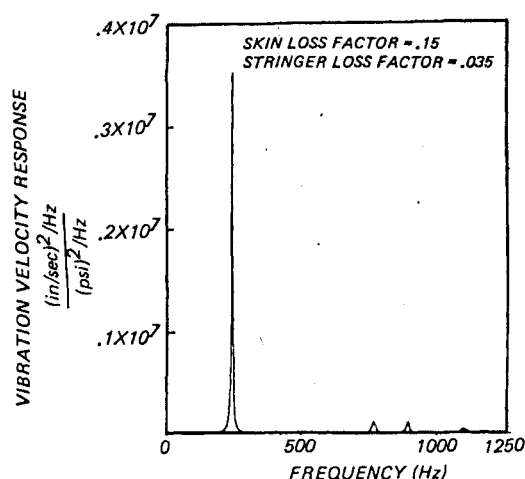


Fig. 8 Predicted effect of skin damping on the response at the skin-bay center (point A, Fig. 1).

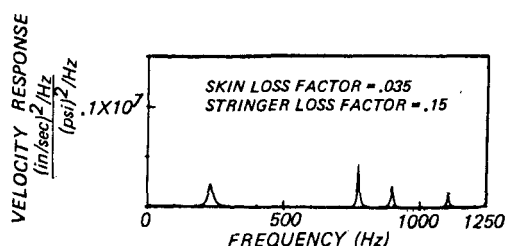


Fig. 9 Predicted effect of stringer damping on the response at the skin-bay center (point A, Fig. 1).

individual skin panel bounded by the stringers and frames. Therefore, application of damping treatment on the skin is very effective in this frequency range. The situation is quite different for the 250 Hz mode. For this mode, the skin, under the influence of in-plane tensile loads, behaves like a stiff member with very little bending action, while being supported on relatively flexible stringers (Fig. 4). As a result, the response of this mode is primarily controlled by stringer resonance, with the skin acting like an attached mass. Because of the relative lack of skin bending, damping treatment on the skin has very little effect at this frequency.

Effect of Stringer Damping

The effect of increasing the stringer loss factor was studied next. Figure 9 shows the predicted response spectrum when the stringer loss factor is increased to 0.15 while skin loss factor is kept at the level of 0.035. It is clearly seen that the 250 Hz mode is now reduced by a significant amount. Comparing with Fig. 3, the reduction of the peak response is seen to be of the order of 13 dB. In an actual structure, the reduction will be different, depending on the correlation properties of the excitation field, the actual level of damping of the baseline structure and the actual boundary conditions of the stringers, and the panel at the frames. If the above narrowband reduction due to stringer damping translates to about 3-5 dB reduction of 250 Hz noise on a broadband basis in an actual fuselage, a significant progress will be made, since the cabin noise in this frequency band is not significantly influenced by skin damping, fiberglass insulation, or lead vinyl.

It may be noticed here that the present situation is analogous to the case of unpressurized skin-stringer structures with closely spaced stringers. In both cases, the fundamental frequency of the individual skin bay is much higher than that of the stringers supported on the frames. As a result, the response at the fundamental mode of the stiffened panel is controlled by the resonance of the stringers, with the skin

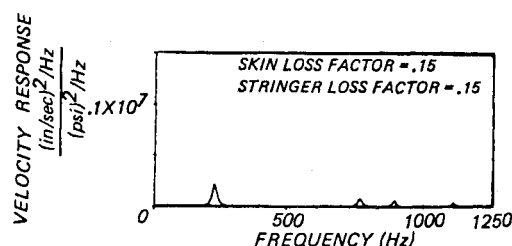


Fig. 10 Predicted effect of skin and stringer damping on the response at the skin bay center (point A, Fig. 1).

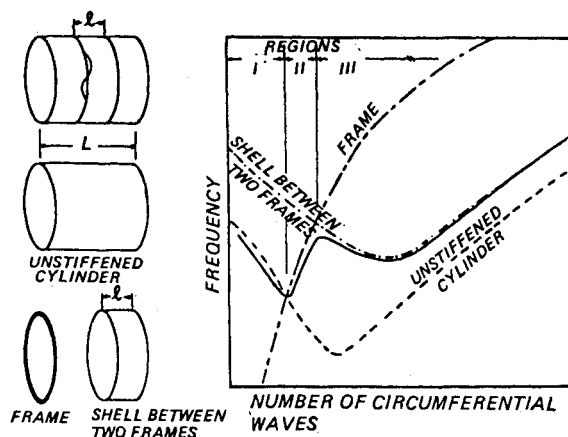


Fig. 11 Natural frequencies of a periodic, frame-stiffened cylinder.

acting like an attached mass, and application of damping on the stringers becomes highly effective. The case of the unpressurized skin-stringer panels was discussed analytically in Ref. 11 and experimentally in Ref. 3, in which it was shown that the response of the panel with 12.7-cm (5-in.) stringer spacing was significantly reduced by stringer damping.

Comparing Fig. 9 with Fig. 3, it is seen that stringer damping also reduces the response of the modes up to about 1000 Hz. Stringer damping is therefore effective in the frequency range 200-1000 Hz, and is particularly effective in the 200-500 Hz range.

Effect of Skin and Stringer Damping

The effect of increasing the skin and stringer damping is shown in Fig. 10. For this calculation, the skin and stringer loss factors were both set at 0.15. Comparing with Fig. 3, it is seen that all the peaks are now reduced to lower levels. Skin damping reduced the peaks in 500-1250 Hz range and stringer damping reduced the response in 200-1000 Hz range. Since the laboratory test results³ show that skin and stringer loss factors of 0.15 can be achieved without a large weight penalty, application of lightweight damping treatment on the skin and stringers seems to be a very efficient way of reducing cabin noise in the low- and midfrequency range. In contrast, the conventional approach of application of local skin damping is ineffective at low frequencies and even at midfrequencies it follows a law of diminishing return beyond a certain point.

Effects of Changing the Various Structural Parameters

The effects of changing the various structural parameters, such as the stringer spacing, stringer stiffness, and skin thickness, were studied next. Only the principal findings will be summarized here. It was observed that the response of the 250 Hz mode was not very sensitive to changes in stringer spacing, stringer stiffness, and the skin thickness. For example, changing the stringer spacing from 22.8 cm (9 in.) to 11.4 cm (4.5 in.) reduced the structural response by about 3 dB. It should be noted that in a real situation, the reduction is expected to be less than what is predicted by this simplified analysis. Increasing the stringer bending stiffness by a factor

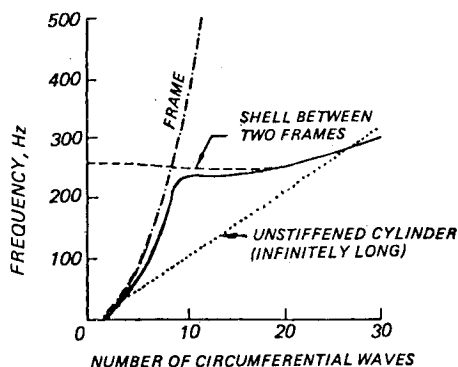


Fig. 12 Natural frequencies of the aft section of a wide-body aircraft, modeled as an infinitely long, frame-stiffened cylinder.

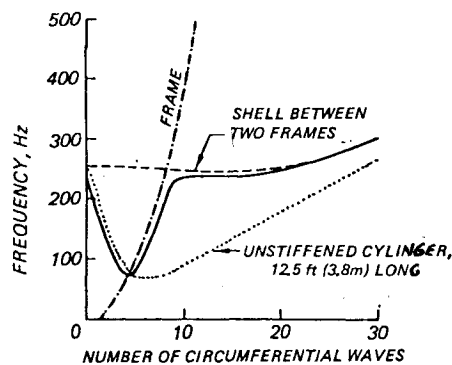


Fig. 13 Natural frequencies of the aft section of a wide-body aircraft, modeled as a finite, frame-stiffened cylinder.

of 5, while keeping the stringer spacing at 22.8 cm (9 in.) reduced the response by about 2 dB. Increasing the skin thickness from 0.16 to 0.25 cm (0.063 to 0.10 in.) with 22.8 cm (9 in.) stringer spacing reduced the response by about 3 dB. Thus the structural response around 250 Hz (or more generally, in the frequency range 200 Hz-500 Hz) is not reduced significantly by increasing the structural stiffness. Because it is also not affected significantly by skin damping, and because this frequency range is below the fundamental frequency (typically 600 Hz) of the individual skin bay carrying in-plane loads due to cabin pressurization, noise transmission in this frequency band had so far been regarded as "stiffness controlled." This study has shown that the structural response in this frequency band is primarily controlled by the stringer resonances, with the skin acting like an attached mass. Application of damping treatment on the stringers should therefore be an effective way of controlling the structural response in the above frequency range.

The increase in skin and stringer damping can be achieved in a number of ways. Any of the following methods or their combinations may be used: 1) unconstrained viscoelastic layer, 2) constrained viscoelastic layers, 3) constrained/unconstrained layer with a spacer mechanism, 4) tuned dampers, 5) tuned foam dampers, 6) laminated skin and stringer (requires adhesives with favorable damping properties), 7) skin and stringers made from composite materials or alloys with high inherent damping.

Effect of Skin and Stringer Damping on the Stress Response and Sonic Fatigue Life of Pressurized Skin-Stringer Structures

From the previous results and discussions, it is apparent that in the 250 Hz mode, the stringer responds very strongly and the panel bends very little. Thus, stringer damping should be very effective in enhancing the stringer sonic fatigue life. It should also be effective in reducing the panel stress response around that frequency. However, the panel stress response was found to be higher in the midfrequency range, due to increased skin bending. Thus skin and stringer damping are both quite effective in reducing the panel and stringer stress response. The optimum combination of skin and stringer damping will be the one for which the skin and the stringers will have an equal sonic fatigue life. This will be a function of the spectrum of excitation, and its correlation and coherence properties.

Effects of Other Complicating Factors

In this analysis, the skin panel and the stringers were assumed to be simply supported at the frame locations. In practice, the frames are shear-tied to the skin in some areas and not so in other areas. The stringers are attached to the frames through clips and are continuous over many frame bays. It is difficult to assess the effects of all these factors. Thus the actual reduction of cabin noise around 250 Hz due to stringer damping will be different from what is predicted by this analysis. The main objective of this work was to identify

the key mechanism of controlling the skin panel motion in the 200-500 Hz range and this is clearly the application of damping treatments on the stringers. The analysis, with all its limitations, has certainly served its purpose, particularly when examined in light of the experimental results shown in Fig. 7. For a more accurate prediction of the effect of stringer damping it will be necessary to consider an excitation model that is more representative of aeroacoustic loading during cruise conditions. It will also be useful to consider a structural model (probably based on a combination of the finite element method and the periodic structure theory^{12,13}) that includes the effects of shear ties, and the continuity of the stringers beyond one frame bay.

V. Reduction of Structural Response Below 250 Hz: Analysis of Frame-Stiffened Cylinders

In the foregoing analysis, the frames were assumed to provide simply supported boundary conditions for the stringers and skin. In reality, the lightweight frames can have considerable response, particularly at frequencies below 250 Hz. To a certain extent the damped stringers should reduce frame and skin vibration through coupling. However, at low frequencies, large scale structural vibrations are involved, and a more detailed analysis is needed.

A preliminary examination of the low-frequency cabin noise problem shows that at low frequencies overall cylindrical modes of fuselage vibrations are involved. In these modes, the fuselage vibrates as a frame-stiffened cylinder (Fig. 2). The core cowl and the fan duct structure of bypass engines also consist of periodically stiffened shells. During takeoff, these structures are subjected to intense fluctuating pressure fields which may eventually cause sonic fatigue cracks in the stiffened skin. Vibration analysis of periodically stiffened shells is therefore also applicable for reducing sonic fatigue of, and case radiation of noise through, propulsion structures.

For this reason, the natural frequencies of a frame-stiffened shell were analyzed using the methods outlined by Hu et al.^{14,15} In these reports, the vibration of a periodically stiffened cylinder was analyzed in terms of the natural frequencies of the various structural components. The effect of internal pressurization was not considered by Hu et al. Including this effect increases the frequencies of the shell, and the following discussion assumes that the cylinder is pressurized. The frequency curve for the stiffened shell, plotted against the number of circumferential waves, is divided into three regions (Fig. 11):

Region I: In this region, the shell dynamics are essentially dominated by those of the unstiffened shell, with frames acting like small, locally attached masses.

Region II: In this region, the shell dynamics are essentially dominated by frame resonances, with the shell segments acting like attached masses. Application of damping treatment on the frames should be useful in reducing structural

$$\nabla^4 w - \frac{N_x}{D_{sk}} \frac{\partial^2 w}{\partial x^2} - \frac{N_y}{D_{sk}} \frac{\partial^2 w}{\partial y^2} - \lambda^4 w = \frac{p(x, y, t)}{D_{sk}}$$

In the following analysis, the skin edges along the frames are considered to be simply supported. The structural response in the longitudinal direction may then be assumed to vary in proportion to $\sin(n\pi y/b)$ in between the frames, where b is the frame spacing. The component of the pressure field convecting in the longitudinal direction may also be decomposed in terms of the modal components varying as $p_n \sin(n\pi/b)$. The solution for the n th mode is then given by

$$w = A_n \exp[\lambda_{1n}x] + B_n \exp[-\lambda_{1n}x] + C_n \exp[-i\lambda_{2n}x] + D_n \exp[-i\lambda_{2n}x] + \frac{p_n \exp[-ik_x x]}{D_{sk}(k_x^2 + \lambda_{1n}^2)(k_x^2 - \lambda_{2n}^2)} \quad (A2)$$

where

$$\lambda_{1n}^2 = \sqrt{\left(\beta^2 + \frac{N_x}{2D_{sk}}\right)^2 - \left(\beta^4 - \lambda^4 + \frac{N_y}{D_{sk}}\beta^2\right) + \left(\beta^2 + \frac{N_x}{2D_{sk}}\right)}$$

$$\lambda_{2n}^2 = \sqrt{\left(\beta^2 + \frac{N_x}{2D_{sk}}\right)^2 - \left(\beta^4 - \lambda^4 + \frac{N_y}{D_{sk}}\beta^2\right) - \left(\beta^2 + \frac{N_x}{2D_{sk}}\right)}$$

$$\beta = \frac{n\pi}{b}, \quad \lambda^4 = \frac{\rho_{sk} h}{D_{sk}} \omega^2, \quad \omega = 2\pi f$$

$$D_{sk} = \frac{E_{sk} h^3}{12(1-\nu^2)} (1 + i\eta_{sk}), \quad N_x = pR, \quad N_y = \frac{pR}{2}$$

The above equation gives the response of the n th mode. The response over a large number of modes is given by the summation over all the modes of interest.

From Eq. (A2) for skin deflection, the equations for the slope, bending moment, and shear force at any point in the panel (for the n th mode) can be derived. Expressed in a matrix form,

$$\{Z\} = [A][E(x)]\{A\} + P e^{-ik_x x} \{K\} \quad (A3)$$

where

$$\{Z\} = \begin{Bmatrix} W \\ \theta \\ M \\ S \end{Bmatrix}, \{A\} = \begin{Bmatrix} A_n \\ B_n \\ C_n \\ D_n \end{Bmatrix}, \{K\} = \begin{Bmatrix} 1 \\ -ik_x \\ -D_{sk} k_x^2 \\ iD_{sk} k_x^3 \end{Bmatrix}$$

$$[A] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ \lambda_{1n} & -\lambda_{1n} & i\lambda_{2n} & -i\lambda_{2n} \\ D_{sk} \lambda_{1n}^2 & D_{sk} \lambda_{1n}^2 & -D_{sk} \lambda_{2n}^2 & -D_{sk} \lambda_{2n}^2 \\ D_{sk} \lambda_{1n}^3 & -D_{sk} \lambda_{1n}^3 & -iD_{sk} \lambda_{2n}^3 & iD_{sk} \lambda_{2n}^3 \end{bmatrix}$$

$$[E(x)] = \begin{bmatrix} \exp[\lambda_{1n}x] & 0 & 0 & 0 \\ 0 & \exp[-\lambda_{1n}x] & 0 & 0 \\ 0 & 0 & \exp[i\lambda_{2n}x] & 0 \\ 0 & 0 & 0 & \exp[-i\lambda_{2n}x] \end{bmatrix}$$

$$P = \frac{p_n}{D_{sk}(k_x^2 + \lambda_{1n}^2)(k_x^2 - \lambda_{2n}^2)}$$

Equation (A1) can now be rewritten as

$$\{Z_2\} = \exp[-ik_x a] \{Z_0\} \quad (A4)$$

From Eq. (A3)

$$\{Z_0\} = [A][E(0)]\{A\} + P\{K\} = [A]\{A\} + P\{K\} \quad (A5)$$

and

$$\{Z_1\} = [A][E(a)]\{A\} + P \exp[-ik_x a] \{K\} \quad (A6)$$

Consideration of continuity across the stringers and the equilibrium of the stringers yields

$$\{Z_2\} = [S]\{Z_1\} \quad (A7)$$

where

$$[S] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & K_R & 1 & 0 \\ -K_T & 0 & 0 & 1 \end{bmatrix}$$

Substituting for $\{Z_2\}$ from Eq. (A4) in Eq. (A7),

$$\exp[-ik_x a] \{Z_0\} = [S]\{Z_1\} \quad (A8)$$

From Eqs. (A5) and (A7) we can substitute for $\{Z_0\}$ and $\{Z_1\}$ in Eq. (A8). We then obtain

$$\exp[-ik_x a] [A]\{A\} + P \exp[-ik_x a] \{K\} = [S][A][E(a)]\{A\} + P \exp[-ik_x a] [S]\{K\}$$

or,

$$([S][A][E(a)] - \exp[-ik_x a][A])\{A\} = P \exp[-ik_x a] ([I] - [S])\{K\} \quad (A9)$$

where $[I]$ is a unit matrix of order 4×4 .

Equation (A9) can now be used to solve for the coefficient matrix $\{A\}$. Substituting for $\{A\}$ in Eq. (A3), any of the response quantities can be calculated.

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